Big Ideas Math: Geometry

Chapter Summary

Chapter 6: Relationships Within Triangles

Core Vocabulary

A point is ***equidistant*** from two figures when the point is the same distance from each figure.

When three or more lines, rays, or segments intersect in the same point, they are called ***concurrent*** lines, rays, or segments.

The point of intersection of concurrent lines, rays, or segments is called the ***point of concurrency***.

The point of concurrency of the three perpendicular bisectors of a triangle is the ***circumcenter*** of the triangle.

The point of concurrency of the angle bisectors of a triangle is the ***incenter*** of the triangle

A ***median of a triangle*** is a segment from a vertex to the midpoint of the opposite side.

The point of concurrency of the three medians of a triangle is called the ***centroid***.

An ***altitude of a triangle*** is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side.

The point of concurrency of the lines containing the altitudes of a triangle is the ***orthocenter*** of the triangle.

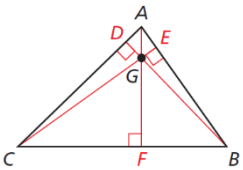
A ***midsegment of a triangle*** is a segment that connects the midpoints of two sides of the triangle.

A style of proof in which you temporarily assume that the desired conclusion is false, then reason logically to a contradiction which proves that the original statement is true is called an ***indirect proof***.

Core Concept

Orthocenter

The lines containing the altitudes of a triangle are concurrent. This point of concurrency is the orthocenter of the triangle.



The lines containing $\overline{AF}$, $\overline{BD}$, and $\overline{CE}$ meet at the orthocenter G of $\triangle ABC$.

How to Write an Indirect Proof (Proof by Contradiction)

Step 1 Identify the statement you want to prove. Assume temporarily that this statement is false by assuming that its opposite is true.

Step 2 Reason logically until you reach a contradiction.

Step 3 Point out that the desired conclusion must be true because the contradiction proves the temporary assumption false.

Learning Goals

Use perpendicular bisectors to find measures.

Use angle bisectors to find measures and distance relationships.

Write equations for perpendicular bisectors.

Use and find the circumcenter of a triangle.

Use and find the incenter of a triangle.

Use medians and find the centroids of triangles.

Use altitudes and find the orthocenters of triangle.s

Use midsegments of triangles in the coordinate plane.

Use the Triangle Midsegment Theorem to find distances.

Write indirect proofs.

List sides and angles of a triangle in order by size.

Use the Triangle Inequality Theorem to find possible side lengths of triangles.

Compare measures in triangles.

Solve real-life problems using the Hinge Theorem.

Segments, Lines, Rays, and Points in Triangles

	Point of Concurrency	Property
Perpendicular Bisector	circumcenter	The circumcenter P of a triangle is equidistant from the vertices of the triangle.
Angle Bisector	incenter	The incenter I of a triangle is equidistant from the sides of the triangle.
Median	centroid	The centroid R of a triangle is two thirds of the distance from each vertex to the midpoint of the opposite side.
Altitude	orthocenter	The lines containing the altitudes of a triangle are concurrent at the orthocenter O.

Theorems

6.1 Perpendicular Bisector Theorem
In a plane, if a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

6.2 Converse of the Perpendicular Bisector Theorem
In a plane, if a point is equidistant from the endpoints of a segment, then it lies on the perpendicular bisector of the segment.

6.3 Angle Bisector Theorem
If a point lies on the bisector of an angle, then it is equidistant from the two sides of the angle.

6.4 Converse of the Angle Bisector Theorem
If a point is in the interior of an angle and is equidistant from the two sides of the angle, then it lies on the bisector of the angle.

6.5 Circumcenter Theorem
The circumcenter of a triangle is equidistant from the vertices of the triangle.

6.6 Incenter Theorem
The incenter of a triangle is equidistant from the sides of the triangle.

6.7 Centroid Theorem
The centroid of a triangle is two-thirds of the distance from each vertex to the midpoint of the opposite side.

6.8 Triangle Midsegment Theorem
The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side.

6.9 Triangle Longer Side Theorem
If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

6.10 Triangle Larger Angle Theorem
If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

6.11 Triangle Inequality Theorem
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

6.12 Hinge Theorem
If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second.

6.13 Converse of the Hinge Theorem
If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second.

Essential Questions

What conjectures can you make about a point on the perpendicular bisector of a segment and a point on the bisector of an angle?

What conjectures can you make about the perpendicular bisectors and the angle bisectors of a triangle?

What conjectures can you make about the medians and altitudes of a triangle?

How are the midsegments of a triangle related to the sides of the triangle?

How are the sides related to the angles of a triangle? How are any two sides of a triangle related to the third side?

If two sides of one triangle are congruent to two sides of another triangle, what can you say about the third sides of the triangles?

Games

- Race for Distance

This is available online in the *Game Closet* at www.bigideasmath.com.

What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations.

The Chapter 6: Building a Roof Truss STEM Video is available online at www.bigideasmath.com.

Additional Review

- Segments, Lines, Rays, and Points in Triangles, *p. 323*
- Using the Midsegment of a Triangle, *p. 330*